Grade 8 Math
Where the Fractions Run Out

42 sheets · every strand · the last year of elementary school

THIS BOOK BELONGS TO

....

Find your strand

Brand new this year

Irrational numbers
not a fraction, ever

The real number system
the line, complete

Scientific notation
very big and very small

Integer × integer
and ÷ too

Binomials
two terms at once

Two-variable data
do they move together?

Scatter plots
strong, weak, positive, negative

Tree diagrams
three events or more

Tessellations
no gaps, no overlaps

Nano to tera
powers of ten you already say

Parallel line angles
eight angles, two sizes

Pythagoras
$a^2 + b^2 = c^2$

Grown-ups: Grade 8 has **44** specific expectations against Grade 7's 52 — fewer, applied to far **`B1.2`** , the real number system, which every secondary **`E2.4`** , the Pythagorean theorem, the single most-used

Ontario 2020 · 44 expectations

A number that is not a fraction

Grade 7 said every number is one integer over another. Here is one that is not — and it is hiding in every square you have ever drawn.

draw it | measure

Find it yourself

Draw a square with sides of exactly 10 cm. Now measure the diagonal — corner to opposite corner — as accurately as you can.

My diagonal: .... cm

You should have got about 14.1 cm. Divide it by 10. That number is $\sqrt{2}$, and here it is written out further than anyone needs:

1.4142135623730950488016887242096980785696718753769 4...

Look for a repeating block in those digits. Circle one if you find it.

Two words

Rational — can be written as one integer over another. Every whole number, every fraction, every decimal that stops or repeats.
Irrational — cannot. Not with a bigger denominator, not ever. The decimal never ends and never repeats.
Together they make the **real number system**.

There is no repeating block, and there never will be. "Irrational" does not mean unreasonable — it means *not a ratio*. That is all.

Grown-ups: drawing the square first matters. $\sqrt{2}$ is not an abstraction — it is a length you can measure with a ruler and can never write down exactly.

Sorting the real numbers

Every number you have ever met is in one of two boxes. Most are in the first.

sort · justify

Number	Rational or irrational?	How you can tell
7		
−3/4		
0.375		
0.333...		
$\sqrt{9}$		
$\sqrt{2}$		
π		
$\sqrt{16}$		
$\sqrt{5}$		

$\sqrt{9}$ and $\sqrt{2}$ behave completely differently. Work out both. What is it about 9 that makes its root rational, and 2 that does not?

The test for a square root: if the number under the sign is a perfect square, the root is rational. If it is not, the root is irrational. There is no in-between.

Grown-ups: 0.333... repeats, so it *is* rational — it is 1/3. Students expect "never-ending" to mean irrational, and this row is the one that fixes it. B1.2 · B1.3

Putting an irrational number somewhere

> $\sqrt{2}$ has a definite place on the line. Finding it without a calculator is the skill.

estimate check

Squeeze it between two whole numbers

$\sqrt{50}$: the nearest perfect squares are 49 and 64. So $\sqrt{50}$ sits between 7 and 8 — and much closer to 7, because 50 is barely past 49.

Root	Between which two whole numbers?	Closer to which?	Your estimate to 1 dp
$\sqrt{20}$			
$\sqrt{30}$			
$\sqrt{75}$			
$\sqrt{110}$			

Mark each of these: $\sqrt{2}$ $\sqrt{8}$ $\sqrt{11}$ π $3/2$

π is about 3.14159... and it is irrational too. It sits between 3 and 4, just past 3.1 — and you have been using it since Grade 6.

Numbers too big and too small to write

Scientific notation. One digit, a point, and a power of ten.

convert · compare

The rule

Move the point until there is exactly **one non-zero digit in front of it**. Count how far you moved. That count is the power.

$4\,250\,000 = 4.25 \times 10^{6}$ · $0.00037 = 3.7 \times 10^{-4}$

Moving left gives a positive power. Moving right gives a negative one.

Written out	Scientific notation
93 000 000	
0.00052	
	6.1×10^{8}
	2.4×10^{-5}
0.000000001	

The trap. Is 3×10^{-4} a negative number? Work it out as an ordinary decimal and say what the minus sign is actually doing.

As a decimal: ....

It is 0.0003 — positive, just very small. The minus sign in the exponent says *small*, not *negative*. Nothing about the number is below zero.

Grown-ups: the negative-exponent trap catches nearly everybody once. Writing it out as a plain decimal settles it permanently.

Squares and roots you should just know

Recall, not calculation. These come up constantly from here on.

recall | test yourself

n	1	2	3	4	5	6	7	8	9	10	11	12	13	15	20
n^2	1	4													

Now cover the table and fill these from memory.

$\sqrt{81}$ = ...

$\sqrt{144}$ = ...

$\sqrt{169}$ = ...

$\sqrt{225}$ = ...

$\sqrt{400}$ = ...

$\sqrt{121}$ = ...

A useful shortcut. $\sqrt{400} = 20$. Explain how knowing $\sqrt{4} = 2$ gets you there without any working.

Every one of these will turn up in Pythagoras on sheet 35. Knowing them cold is the difference between solving a triangle and getting stuck halfway.

Why a negative times a negative is positive

Nobody should just be told this. Find the pattern and let it force the answer.

find the pattern | 8 continue it

Calculation	Answer
$3 \times (-4)$	-12
$2 \times (-4)$	-8
$1 \times (-4)$	-4
$0 \times (-4)$	0
$-1 \times (-4)$	
$-2 \times (-4)$	
$-3 \times (-4)$	

Look down the answer column. It goes −12, −8, −4, 0 — rising by 4 each time. If the pattern keeps going, what has to come next? And what does that force?

The rule the pattern gives you

Same signs → positive. (+)(+) = + and (−)(−) = +
Different signs → negative. (+)(−) = − and (−)(+) = −
Division works exactly the same way.

The pattern forces it. You did not have to be told, and a rule you derived is a rule you can rebuild if you forget it.

Grown-ups: this is worth ten minutes. Students told "two negatives make a positive" apply it to addition; students who built the pattern do not.

All four operations, below zero

Adding, subtracting, multiplying and dividing. Watch which rule you are using.

calculate | check signs

$-7 + 12 =$ ....

$5 - (-8) =$ ....

$-4 - 7 =$ ....

$-15 + (-5) =$ ....

$-6 \times 4 =$ ....

$-9 \times (-3) =$ ....

$-36 \div 9 =$ ....

$-48 \div (-6) =$ ....

The trap that catches everybody. Two of these look almost identical and have completely different answers:

$-6 + (-4) =$ .... $-6 \times (-4) =$ ....

Explain in one sentence why the "two negatives make a positive" rule applies to one and not the other.

It is a rule about multiplying and dividing, not adding. Two negatives added together just go further down: −10.

Mixed numbers, all four ways

Grade 7 did fraction by fraction. Grade 8 adds whole numbers and mixed numbers to the mix.

convert first | calculate

Always convert the mixed number first

$2\frac{1}{2} \div \frac{3}{4}$. Turn $2\frac{1}{2}$ into **5/2** before you do anything else. Then flip and multiply: $5/2 \times 4/3 = 20/6 =$ **10/3**, or $3\frac{1}{3}$.

Trying to flip a mixed number directly produces confident nonsense.

$1\frac{1}{2} \times \frac{2}{3} =$ ....

$3\frac{1}{4} \div \frac{1}{2} =$ ....

$\frac{4}{5} \div 2 =$ ....

$2\frac{1}{3} \times 3 =$ ....

$\frac{5}{6} + 2\frac{1}{4} =$ ....

$4\frac{1}{2} - 1\frac{2}{3} =$ ....

Predict before you calculate. Will $3\frac{1}{4} \div \frac{1}{2}$ be bigger or smaller than $3\frac{1}{4}$? Say why, then check.

Dividing by a half doubles it. How many halves fit into $3\frac{1}{4}$? Six and a half of them.

Beyond 100% and below 1%

A percent is not capped at 100, and it does not stop at whole numbers either.

convert interpret

Percent	Decimal	Fraction	What it means in words
150%			one and a half times
250%			
0.5%			
0.1%			
	3.5		

Work these out

150% of 80 = ...

0.5% of 4 000 = ...

200% of 35 = ...

0.1% of 50 000 = ...

A headline says a town's population grew 300%. It was 2 000 before. What is it now — 6 000 or 8 000? Be careful.

Grew *by* 300% means it added three times itself — so 8 000. Grew *to* 300% would be 6 000. One small word, four thousand people.

Grown-ups: "grew by" versus "grew to" is a real ambiguity in news writing, and noticing it is exactly the kind of thing `B1.4` is for.

Sliding the point

Multiplying and dividing by powers of ten does not need any working at all.

8 mental math | notice

Start	× 10	× 1000	÷ 10	÷ 1000
4.75				
0.062				
380				

What is really happening

The digits never change. Only their **place value** does. × 10 moves every digit one column to the left; ÷ 10 moves it one to the right.
× 1000 is three columns. ÷ 1000 is three the other way. That is all a power of ten ever does.

Say it properly. Somebody tells you the rule is "add a zero". Give one number where that advice fails.

"Add a zero" breaks on decimals. 4.75 × 10 is 47.5, not 4.750. The digits slide; nothing gets added.

Order of operations, with everything in

Rational numbers, ratios, rates and percents in the same problem. Multiple steps.

evaluate check

Two ranks, not six steps

1. Brackets 2. Exponents
3. × and ÷ — same rank, left to right 4. + and − — same rank, left to right

$-8 + 3 \times (-2) =$ ....

$(-4)^2 - 10 =$ ....

$24 \div (-6) \times 2 =$ ....

$-3 \times (5 - 9) =$ ....

$\frac{1}{2} + \frac{3}{4} \times 2 =$ ....

$20 - 4^2 \div 2 =$ ....

$(-4)^2$ and -4^2 are not the same. Work out both and explain what the brackets change.

$(-4)^2 =$ .... $-4^2 =$ ....

$(-4)^2 = 16$ but $-4^2 = -16$. Without brackets, the exponent binds to the 4 and the minus sign is applied afterwards.

Grown-ups: the bracket distinction is genuinely subtle and appears on every secondary test. Worth its own five minutes.

Comparing two proportional situations

Grade 7 asked whether something was proportional. Grade 8 puts two proportional situations side by side and asks which is better value.

8 find the unit rate | compare

Option A	Option B	Unit rate A	Unit rate B	Better value
5 for $6.25	8 for $9.60			
240 km on 20 L	350 km on 28 L			
3 kg for $7.50	500 g for $1.30			

Careful with the third row — the units do not match until you make them match.

Now find the unknown value.

3 : 12 = 7 : ....

5 : .... = 20 : 36

If 4 kg costs $18, then 10 kg costs ....

If 6 items take 15 min, 22 items take ....

Always convert to the same unit first. $1.30 per 500 g is $2.60 per kg, which is more than $2.50 per kg. The smaller price is the worse deal.

Number — how am I doing?

Tick honestly. A blank is information, not a failure.

tick

- [ ] I know what "irrational" actually means
- [ ] I know 0.333… is rational
- [ ] I can write a number in scientific notation
- [ ] I know my squares to 20^2
- [ ] I know $-6 + (-4)$ and $-6 \times (-4)$ are different
- [ ] I can handle percents over 100 and under 1
- [ ] I compare two options using unit rates
- [ ] I can tell a rational number from an irrational one
- [ ] I can place $\sqrt{20}$ between two whole numbers
- [ ] I know 3×10^{-4} is positive
- [ ] I can explain why $(-3)(-4) = +12$
- [ ] I convert a mixed number before dividing
- [ ] I know $(-4)^2$ and -4^2 are different

The one I want to get better at is…

…and the sheet that would help is number ……

Grown-ups: rows one, eight and nine are the ones to protect. Everything in Grade 9 assumes all three.

B1.1–B2.8

Patterns that shrink

Grade 7 compared two growing patterns. Grade 8 handles a negative rate exactly the same way.

find the rate | write the rule

Pattern	Initial value	Constant rate	Rule	Term 10
40, 34, 28, 22, ...				
2.5, 4.0, 5.5, 7.0, ...				
−8, −5, −2, 1, ...				
100, 87.5, 75, 62.5, ...				

Two water tanks. Tank A holds 200 L and loses 12 L an hour. Tank B holds 140 L and loses 4 L an hour. Which empties first, and when are they equal?

Equal after .... hours, at .... litres

A shrinking pattern is still linear. The constant rate is just negative — which is a negative slope, a year before that word arrives.

Patterns with rational numbers

Fractions, decimals and negatives — every kind of **rational number** — inside the same pattern. Nothing about the method changes.

8 extend

work backwards

Term	1	2	3	4	5	n
Pattern A	½	1¼	2			
Pattern B	−3	−1.5	0			
Pattern C	12	9.5	7			

Work backwards. Which term of Pattern C is the first to go below zero? Solve it, do not list.

Make your own. A shrinking pattern with a constant rate of −¾ that starts at 5.

Pattern B crosses zero exactly. Not every pattern does — whether it lands on zero or steps over it depends on the initial value and the rate together.

Adding two terms at once

A **binomial** is two terms joined by a plus or a minus. Adding two of them just means collecting the like terms.

collect | check signs

$(3x + 5) + (2x - 8)$

The x terms go together: $3x + 2x = 5x$. The plain numbers go together: $5 + (-8) = -3$.

$= 5x - 3$

x terms and plain numbers never combine. $5x - 3$ is as far as it goes.

$(4a + 2) + (3a + 7) =$ ....

$(6n - 1) + (-2n + 9) =$ ....

$(5y + 3) + (-5y - 3) =$ ....

$(-2p + 8) + (7p - 12) =$ ....

$9m - 4m =$ ....

$3k - 11k =$ ....

The third one came to zero. What does that tell you about those two binomials?

They are opposites. Every term in one is the negative of the matching term in the other, so they cancel completely.

Grown-ups: the integers inside the brackets are where marks go, not the algebra. $5 + (-8)$ is Grade 7 work arriving inside a Grade 8 skill.

Substituting rational numbers

Fractions, decimals and negatives going into the same expression. Multiply first, then add.

substitute | check

Expression	n = 4	n = 0.5	n = −3	n = −½
$5n - 2$				
$10 - 4n$				
$n^2 + 1$				
$-2n + 6$				

Look at the n^2 row with n = −3. Is the answer 10 or −8? Explain which and why, using sheet 11.

It is 10. n^2 means n × n, so (−3)(−3) = +9, then +1. The negative disappears because it is being multiplied by itself.

Grown-ups: squaring a negative substituted value is the classic Grade 8 slip, and it connects straight back to the bracket question on sheet 11.

Solving with integers, and verifying

Undo in reverse. Then put your answer back in — the expectation says *verify*, and it means it.

solve | verify

Equation	n =	Check — substitute and write it out
$3n + 7 = -8$		
$-4n = 20$		
$2n - 9 = -15$		
$5n + 3n = -24$		
$1.5n + 2 = -4$		
$-2n + 5 = 11$		

Two of these have negative answers and two have positive ones. Before you check, could you tell which from the equation alone? Say what you looked at.

Verifying is half the expectation. An answer with no substitution back is a guess that happens to be written neatly.

The inequality that flips

One rule in all of Grade 8 algebra behaves differently from everything you have learned. This is it.

solve | test it

Watch what happens

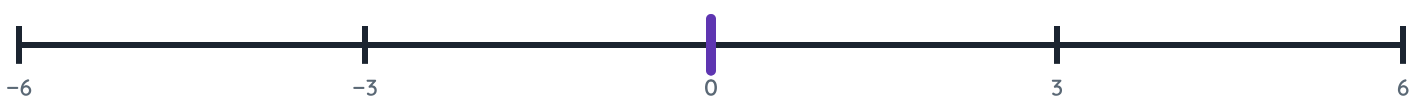
Start with something true: **4 > 2**. Now multiply both sides by **−1**.

−4 > −2 is that still true?

No. −4 is further left on the number line, so −4 < −2. Multiplying by a negative **reversed the direction.**

Inequality	Solution	Did the sign flip? Why?
$3n + 4 < 19$		
$-2n < 6$		
$-5n + 3 \geq 18$		
$4n - 7 > -19$		

Graph the second one. Shade every integer that works.



Test your shading. Pick one number you shaded and one you did not. Substitute both into $-2n < 6$. Does each behave as it should?

$-2n < 6$ gives $n > -3$, not $n < -3$. Substituting is the only way to be certain, and it takes ten seconds.

Grown-ups: this rule reappears throughout secondary mathematics and is almost always taught as a bare instruction. The 4 > 2 demonstration is why it happens.

Code that helps you decide something

This is where all eight years of coding have been heading. The program reads data and tells somebody what to do.

write it | 8 trace it

Analysing data to inform a decision

```
temperatures = [18, 22, 25, 19, 31, 28, 24]
total = 0
hot_days = 0
for each t in temperatures:
  total = total + t
  if t > 25:
    hot_days = hot_days + 1
mean = total / 7
if hot_days > 2: print "Open the pool early"
else: print "Normal opening"
```

Trace it by hand

total = ...

hot_days = ...

mean = ...

It prints: ....

Change one number. What is the smallest change to the temperature list that would flip the decision?

Now write your own. A program that reads a list of your marks and prints either "on track" or "needs work". Say what threshold you chose and why.

Notice the program does not just calculate — it recommends. And somebody had to choose the threshold. Change 25 to 20 and the same data gives the opposite advice.

Grown-ups: "somebody chose the threshold" is the most important idea on this sheet, and it is the honest bridge to how real automated decisions are made.

C3.1 · C3.2

One variable, or two?

Every year until now has handled one thing at a time. Grade 8 handles two, and asks whether they move together.

classify explain

Question	One variable or two?	What you would record
How tall is everyone in the class?		
Does height relate to shoe size?		
What is the most common eye colour?		
Do students who sleep more score higher?		
How much rain fell each month?		
Does temperature affect ice cream sales?		

The distinction

One variable — you measure one thing about many people or times. Answers "how much" or "how many".
Two variables — you measure *two things about the same person or moment*, then ask whether they are connected.

The giveaway word. Look at the two-variable questions. What word or phrase do they all contain that the one-variable questions do not?

They all ask about a relationship — "does X relate to Y", "do students who... score higher", "does X affect Y". One variable describes. Two variables connect.

Grown-ups: this is the biggest conceptual jump in the Grade 8 data strand, and the pairing must be per-person: *this* student's sleep with *this* student's score.

D1.1 · D1.2

Plotting a cloud of points

A scatter plot puts one variable along the bottom and the other up the side. Each point is one person, one moment, one thing.

plot

look at the shape

Hours of sleep	5	6	6	7	7	8	8	9	9	10
Test score	52	58	61	65	71	74	78	80	85	83

Label the bottom axis **hours of sleep, 4 to 11** and the side axis **score, 40 to 90**. Then plot all ten points.

Describe the cloud. Does it tilt up, tilt down, or sit flat?

Is it tight or loose? Could you draw a single straight line that most points sit near?

Do not join the dots. A scatter plot is not a line graph. Each point is a separate person, and there is nothing in between them.

Grown-ups: joining scatter-plot points is the commonest error, and it comes from four years of broken-line graphs. Worth naming.

D1.2 · D1.3

Five words for a relationship

Strong or weak. Positive or negative. Or none at all. Those five words are the whole expectation.

read the shape | name it

The vocabulary

Positive — as one goes up, so does the other. The cloud tilts up to the right.
Negative — as one goes up, the other goes down. The cloud tilts down.
Strong — the points sit close to a line. **Weak** — they scatter widely but still tilt.
None — no tilt at all. Just a cloud.

Two things measured together	Which relationship would you expect?
Hours of practice and skill level	
Outside temperature and heating costs	
Shoe size and test score	
Age of a car and its resale price	
Height and arm span	
Number of pets and hours of sleep	

Now add one outlier. Go back to your sleep-and-score plot and add a point at (10, 45). What happens to how strong the relationship looks?

Height and arm span is the strongest one here — for most people they are almost equal. That is what a strong positive relationship looks like.

Together is not because

The most useful sheet in this book. Two things can move together perfectly and have nothing to do with each other.

think | find the third thing

The classic one

Ice cream sales and drowning rates rise and fall together — strongly and positively. Does ice cream cause drowning?

No. Hot weather causes both. It is a **third thing** driving them, and it is not on the graph.

Two things that rise together	What third thing might explain it?
Shoe size and reading ability, in children	
Number of firefighters at a fire and damage caused	
Hours of study and test score	
Number of hospitals in a city and number of illnesses	

The third row is different from the others. Studying probably *does* cause a better score. So how would you ever tell the difference between a real cause and a third thing?

A scatter plot can never tell you. It shows that two things move together and stops there. To find a cause you have to change one thing on purpose and hold the rest still — which is exactly what an experiment is.

Grown-ups: the curriculum asks for the *relationship*, never the *cause*. This distinction is the single most useful piece of adult data literacy in the elementary programme.

D1.5 · D1.6

Choosing the graph, and making the infographic

Eight years of graph types. Now you have to pick the right one and say why.

choose | make

The data	Best graph	Why
Height and arm span for 30 people		
How a class of 30 splits between four sports		
Daily temperature across one month		
Heights of everyone in school, grouped in bands		
How many students play each of five sports		

Your options: scatter plot · circle graph · broken-line graph · histogram · bar graph

Make an infographic. Collect two variables from ten people. Include a table, a scatter plot, the relationship in words, and one thing your graph cannot tell you.

"One thing your graph cannot tell you" is the line most people leave off. It is also the line that makes the rest believable.

Three events, and a tree

Two events you can do in your head. Three or more, you draw the tree.

draw the tree | 8 calculate

How a tree diagram works

Every branch is one choice. **Multiply along a branch** to get the chance of that whole path.
Add across branches that all count as a win. Every path is on the tree, so nothing gets missed.

Three coin flips. Draw the whole tree. There are eight paths.

P(three heads) = ... P(exactly two heads) = ... P(at least one tail) = ...

Now a dependent one. A bag holds 3 red and 5 blue. You draw three marbles and keep each one. What is the chance all three are blue?

.... × .... × .... = ...

Why does the bottom number change each time?

"At least one tail" is the fast one. Instead of adding seven paths, work out P(no tails) and subtract from 1. Same answer, one line.

Grown-ups: the "at least one" complement trick generalises to everything and is worth teaching as a named move.

Shapes that tile without gaps

A shape tessellates if copies of it cover a surface with no gaps and no overlaps. Some do. Most do not — and the reason is angles.

tile it | 8 work out why

The test

At every point where corners meet, the angles must add to exactly **360°**. Not 350, not 370.

A square's corner is 90°. Four of them make 360°. It tessellates.

Regular shape	Each interior angle	360 ÷ that angle	Whole number? Tessellates?
Equilateral triangle	**60°**		
Square	**90°**		
Regular pentagon	**108°**		
Regular hexagon	**120°**		
Regular octagon	**135°**		

Make a tessellation from one of the three that work. Colour it.

Which transformations can you find in your tessellation — translation, reflection, rotation? Name them and say where.

Only three regular shapes tessellate on their own: the triangle, the square and the hexagon. Everything else leaves a gap, and 360 ÷ 108 = 3.33... is exactly why the pentagon fails.

Grown-ups: `E1.1` asks for the **transformations** in the tessellation, not just the tiling. Bees use hexagons because they tile with the least wall material. E1.1

Building from the plan

Grade 7 drew the views from the object. Grade 8 does it backwards — which is what anybody reading a plan actually has to do.

build | scale

top | front | side

Get somebody to draw three views of an object without telling you what it is. Then build it from cubes or card.

What I built

Was it what they had in mind? ....

Three views are not always enough. Sketch two different objects that would give the *same* three views. What extra information would a builder need?

This is a real limitation, not a trick question. It is why architectural plans include sections and notes, not just three views.

Grown-ups: the ambiguity question is the interesting part of `E1.2` and rarely gets asked. Hidden internal cubes are the usual answer.

Scale drawings, and the area trap

Double every length and the area does not double. It quadruples. This catches adults.

calculate redraw

Reading a scale

A plan says **1 : 50**. A wall measures 8 cm on the plan. In real life it is 8 × 50 = 400 cm = **4 m**.

Scale	On the plan	In real life
1 : 50	12 cm	
1 : 100	6.5 cm	
1 : 25		3 m
1 : 200		15 m

The area trap. A room is 4 cm × 3 cm on a 1 : 50 plan.

Area on the plan: .... cm^2

Real dimensions: .... m by .... m

Real area: .... m^2

The lengths were multiplied by 50. What were the areas multiplied by?

Redraw at a different ratio. Take your 1 : 50 plan and redraw it at 1 : 100. What happens to every measurement?

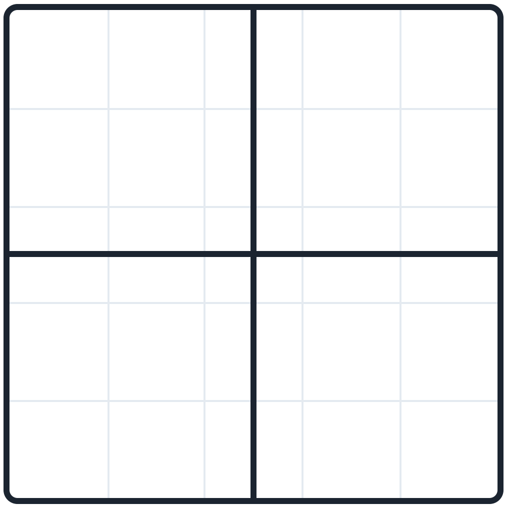
Areas scale by the square of the ratio. Lengths × 50 means areas × 2 500. Same trap as doubling a pizza's radius in Grade 7.

Grown-ups: the length-versus-area scaling error is extremely common in adults reading floor plans and property listings.

Four transformations on the plane

> Slide, flip, turn and resize. Three keep the size; one does not.

plot | 8 predict



Label both axes −6 to +6. Plot triangle A(1,1), B(3,1), C(1,2).

Transformation	A →	B →	C →
Reflect in the y-axis			
Rotate 180° about (0,0)			
Translate 2 left, 4 down			
Dilate by scale factor 2			

Three of those four produce a congruent image. One does not. Which, and what exactly did it change and keep?

The dilation is the odd one out. It changes every length and keeps every angle — which makes the image *similar*, not congruent.

Nano to tera

You already say most of these words. Every one of them is a power of ten.

8 convert | connect

Prefix	Symbol	Power of ten	Written out	Somewhere you have met it
tera	T	10^{12}		
giga	G	10^{9}		a gigabyte
mega	M	10^{6}		
kilo	k	10^{3}	1 000	a kilometre
milli	m	10^{-3}	0.001	a millimetre
micro	μ	10^{-6}		
nano	n	10^{-9}		a nanometre
pico	p	10^{-12}		

Convert

2 GB = ... bytes

500 nm = ... m

3 MW = ... W

40 μm = ... m

How many nanometres in a millimetre? Work it out from the powers, not by guessing.

A human hair is roughly 80 000 nanometres across. A cell is around 20 000. That is the scale your microscope work in Science lives at.

Grown-ups: connecting "gigabyte" to 10^9 is the whole point — students use these words daily without ever meeting them as mathematics.

Eight angles, two sizes

One straight line crossing two parallel lines makes eight angles. Only two of them are actually different.

measure | name the rule

Draw it. Two parallel lines, then one straight line crossing both at a slant. Number all eight angles.

Measure them all. How many different sizes did you find? ....

The three angle families

Corresponding — same position at each crossing. **Equal.** They make an F shape.
Alternate — opposite sides of the crossing line, between the parallels. **Equal.** A Z shape.
Co-interior — same side, between the parallels. They **add to 180°**. A C shape.

Given	Find	Which rule?
One angle is 65°. Its corresponding angle is…		
One angle is 65°. Its co-interior angle is…		
One angle is 112°. Its alternate angle is…		
A pentagon's interior angles add to…		

Every answer needs the rule attached. "65° because corresponding angles are equal" is a complete answer. Just the number is not.

Grown-ups: the F, Z and C shapes are a reliable memory aid. Interior angles of an n-sided polygon total (n − 2) × 180°.

Three squares on three sides

The most-used result in all of secondary mathematics, and it is a statement about **areas** — which is why it is a^2, not a.

build it | 8 add the areas

Do this before you use any formula

On squared paper, draw a right triangle with legs of **3** and **4**. Now build an actual square on each of the three sides — 3×3, 4×4, and one on the slanted side.

Count the squares inside each one.

Draw it here

Small square: .... Medium square: .... Large square: ....

Do the first two add up to the third? ....

What you just proved

In any right triangle, the square on the **hypotenuse** — the longest side, always opposite the right angle — equals the sum of the squares on the other two.
$a^2 + b^2 = c^2$, where c is always the hypotenuse.

9 + 16 = 25. The formula is not a rule somebody invented — it is a fact about areas, and you just counted it.

Using it to find a missing side

Two sides known, one missing. Find the hypotenuse by adding; find a leg by subtracting.

calculate | check it looks right

Finding the hypotenuse — add

$a^2 + b^2 = c^2$

legs 6 and 8 → c = ...

legs 5 and 12 → c = ...

legs 9 and 12 → c = ...

Finding a leg — subtract

$a^2 = c^2 - b^2$

c = 13, b = 5 → a = ...

c = 25, b = 24 → a = ...

c = 10, b = 6 → a = ...

A ladder problem. A 5 m ladder leans against a wall with its foot 1.5 m out. How far up the wall does it reach? Draw it first.

Answer: .... m

Always check it looks right. Somebody says the hypotenuse of a triangle with legs 6 and 8 is 7. Without calculating, how do you know that is wrong?

The hypotenuse is always the longest side. An answer smaller than either leg is wrong before you check the arithmetic.

Shapes made of other shapes

Composite objects. Cut them into pieces you already know, work each one out, then add — or subtract.

decompose | 8 total

Everything you need

rectangle = l × w · triangle = b × h ÷ 2 · circle: area = πr^2, circumference = $2\pi r$
prism or cylinder: **volume = area of base × height** · surface area = add up every face

Composite object	Cut it into...	Answer
An L-shaped room, 6 m × 4 m with a 2 m × 2 m corner removed — **area**		
A rectangle 10 cm × 6 cm with a semicircle of diameter 6 cm on one end — **area**		
A cylinder radius 3 cm, height 10 cm, sitting on a cube of side 8 cm — **volume**		
A running track: rectangle 80 m × 60 m with a semicircle at each end — **perimeter**		

The last one has a trap. When you add the two semicircles' curved edges to the rectangle's sides, which sides of the rectangle do you leave out, and why?

You leave out the two ends — they are inside the track, not on its edge. Drawing the outline and tracing round it with a finger settles it.

Grown-ups: the running-track perimeter is the classic composite question, and the "which edges are actually on the outside" step is where it is won or lost.

E2.3

Spatial Sense — how am I doing?

Tick honestly. A blank is information.

tick

- ☐ I know why a pentagon does not tessellate
- ☐ I can build an object from its three views
- ☐ I can read and use a scale like 1 : 50
- ☐ I can dilate a shape on the Cartesian plane
- ☐ I can name corresponding, alternate and co-interior angles
- ☐ I can explain Pythagoras with squares, not just the formula
- ☐ I check that the hypotenuse is the longest side
- ☐ I can find the transformations in a tessellation
- ☐ I know three views can be ambiguous
- ☐ I know areas scale by the *square* of the ratio
- ☐ I know giga is 10^9 and nano is 10^{-9}
- ☐ I attach the rule to every angle answer
- ☐ I can find a missing leg as well as a hypotenuse
- ☐ I can break a composite shape into known pieces

The one I want to get better at is…

…and the sheet that would help is number ....

Grown-ups: "I can explain Pythagoras with squares" is the row that separates Level 4 from Level 3 in this strand.

Simple against compound

Same money, same rate, ten years. One line stays straight and the other bends upward. Watch the gap open.

8 calculate | graph both

The difference in one line

Simple: interest is always worked out on the original amount.
Compound: interest is worked out on whatever is there *now*, including last year's interest.

Year	Simple at 5% on $1 000	Compound at 5% on $1 000	Gap
1	$1 050.00	$1 050.00	$0
2			
3			
5			
10			

A spreadsheet is on-curriculum here — `F1.4` says *using digital tools*.

Why are year 1 identical and year 10 far apart? Explain what compounding is actually doing.

Compounding works in both directions. It is the best thing about saving and the worst thing about debt — and it is the same arithmetic either way.

A plan that accounts for tax

Grade 7 tracked a budget. Grade 8 adds the thing that surprises everybody the first time they earn money.

plan | calculate

Two kinds of tax you will meet. Sales tax is added at the till — in Ontario, 13% HST on most things. **Income tax and deductions** come off your pay before you ever see it, so what you earn and what arrives are different numbers.

Price before tax	13% HST	Total
$40.00		
$129.99		
		$56.50

The last row works backwards. Divide by 1.13.

My long-term plan

Goal: .... Cost including tax: ....

Month	Money in	Money out	Saved	Running total
1				
2				
3				
4				

A goal priced without tax is short by 13%. On a $400 goal that is $52 — about a month of saving, missing from your plan.

Grown-ups: `F1.2` names *tax implications* explicitly. Pricing a goal before tax is the mistake the expectation exists to prevent. F1.2 · F1.3

Reading a credit card offer

Three numbers decide whether a card is a good deal, and the advertisement will show you only one of them.

compare | 8 work it out

Card	Interest rate	Annual fee	Rewards
Card A	**19.99%**	**$0**	1% back
Card B	**22.99%**	**$120**	4% back
Card C	**12.99%**	**$39**	none

Someone who spends $500 a month and pays the balance in full every time. Which card, and how much do they gain in a year?

Someone who carries a $2 000 balance all year. Which card now, and how much does it cost them?

Same three cards, opposite answers. What single fact about the person decides it?

Also worth knowing. Paying the *minimum* on a credit card is a real trap. On a $2 000 balance at 20%, a 2% minimum payment takes over **twenty years** to clear — and costs far more in interest than the original purchase.

Rewards only beat interest if you never pay interest. Card B's 4% is excellent for one person and expensive for another, with no change to the card at all.

Grown-ups: `F1.6` names credit cards specifically. The two-scenario structure is what makes the comparison meaningful rather than a ranking. F1.6

Getting more for the same money

Sales, loyalty points and incentive programs. They genuinely do return value — and they exist for a reason.

calculate | weigh it

The offer	What you actually save	What it needs you to do
30% off, then 10% off the reduced price		
Buy 2, get 1 free		
1 point per dollar, 100 points = $1		
Free shipping over $75		

30% then 10% is not 40% off. Work it out on a $100 item and say what the real discount is.

After 30%: .... After a further 10%: ....

Real total discount: ....

Both sides. Name one real benefit of a loyalty program to *you*, and one real benefit to *the company* that is not just your money.

30% then 10% is 37% off, not 40%. The second discount applies to the already-reduced price — the same "percent of what?" question from Grade 7.

Money that crosses a border

Exchange rates again — but now with the fees the rate does not show you.

8 convert | find the hidden cost

Amount	Rate	Converted
$300 CAD	**1 CAD = 0.73 USD**	
€250	**1 CAD = 0.68 EUR**	
$500 CAD	**1 CAD = 110 JPY**	

The part the rate hides. You buy something for $100 USD with a Canadian card. The bank uses a rate of 0.71 instead of the market 0.73, and adds a 2.5% foreign transaction fee.

At the market rate it would cost .... CAD

At the bank's rate: .... CAD

Plus the 2.5% fee: .... CAD

Total extra cost, and as a percentage:

Method of paying abroad	An advantage	A disadvantage
Cash exchanged before you go		
Credit card		
A card with no foreign fee		

The advertised rate is never the rate you get. The spread and the fee together are the real cost, and neither appears in the headline number.

What you will actually use

This is the last financial literacy sheet of elementary school. Everything here is real.

tick discuss

- ☐ I know the difference between simple and compound interest
- ☐ I add tax when I price something
- ☐ I know rewards only beat interest if you clear the balance
- ☐ I know 30% then 10% is not 40%
- ☐ I can track income and spending in a table
- ☐ I know compounding works on debt too
- ☐ I can work backwards from a price including tax
- ☐ I know a minimum payment is a trap
- ☐ I know the advertised exchange rate is not the one I get
- ☐ I can say what a loyalty program gives the company

One question to ask an adult you trust. Pick something on this sheet you want to know more about, and write the question you would ask.

Nobody is born knowing any of this. Most adults learned it by getting it wrong once, expensively. You are getting it eight years early and for free.

Grown-ups: the last box is an invitation. A real conversation about a real family financial decision is worth more than any worksheet.

F1.1–F1.6

Answers — Number

Only questions with one right answer are here. Anything asking you to explain or decide is judged on the reasoning.

1. The diagonal of a 10 cm square is about **14.1 cm**, so the ratio is about 1.41. There is **no repeating block** in $\sqrt{2}$ and there never will be — that is what makes it irrational.

2. Rational: 7, −3/4, 0.375, 0.333… (it repeats, so it is 1/3), $\sqrt{9} = 3$, $\sqrt{16} = 4$. Irrational: $\sqrt{2}$, π, $\sqrt{5}$. **The test:** if the number under the root sign is a perfect square, the root is rational.

3. $\sqrt{20}$ is between **4 and 5**, closer to 4 (≈4.5). $\sqrt{30}$ is between **5 and 6**, closer to 5 (≈5.5). $\sqrt{75}$ is between **8 and 9**, closer to 9 (≈8.7). $\sqrt{110}$ is between **10 and 11**, closer to 10 (≈10.5).

4. 93 000 000 = **9.3×10^7** · 0.00052 = **5.2×10^{-4}** · 6.1×10^8 = **610 000 000** · 2.4×10^{-5} = **0.000024** · 0.000000001 = **1×10^{-9}**. 3×10^{-4} = **0.0003**, which is positive — the minus sign means *small*.

5. Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 225, 400. $\sqrt{81} = 9$ · $\sqrt{144} = 12$ · $\sqrt{169} = 13$ · $\sqrt{225} = 15$ · $\sqrt{400} = 20$ · $\sqrt{121} = 11$. $\sqrt{400} = 20$ because 400 = 4 × 100, and $\sqrt{4} \times \sqrt{100} = 2 \times 10$.

6. −1 × (−4) = **+4** · −2 × (−4) = **+8** · −3 × (−4) = **+12**. The answer column rises by 4 every row, and continuing that pattern forces the positives.

7. −7 + 12 = **5** · −6 × 4 = **−24** · 5 − (−8) = **13** · −9 × (−3) = **27** · −4 − 7 = **−11** · −36 ÷ 9 = **−4** · −15 + (−5) = **−20** · −48 ÷ (−6) = **8**. And the trap: −6 + (−4) = **−10**, but −6 × (−4) = **+24**. The sign rule is about multiplying and dividing only.

8. 1½ × ⅔ = **1** · 3¼ ÷ ½ = **6½** · ⅘ ÷ 2 = **2/5** · 2⅓ × 3 = **7** · ⅚ + 2¼ = **3 1/12** · 4½ − 1⅔ = **2 5/6**. Dividing by a half doubles it.

9. 150% = 1.5 = 3/2 · 250% = 2.5 = 5/2 · 0.5% = 0.005 = 1/200 · 0.1% = 0.001 = 1/1000 · 3.5 = 350% = 7/2. 150% of 80 = **120** · 0.5% of 4 000 = **20** · 200% of 35 = **70** · 0.1% of 50 000 = **50**. Grew *by* 300% means it added three times itself: **8 000**.

10. 4.75 → 47.5, 4750, 0.475, 0.00475. 0.062 → 0.62, 62, 0.0062, 0.000062. 380 → 3800, 380000, 38, 0.38. "Add a zero" fails on any decimal — 4.75 × 10 is 47.5, not 4.750.

11. −8 + 3 × (−2) = **−14** · $(-4)^2 - 10$ = **6** · 24 ÷ (−6) × 2 = **−8** · −3 × (5 − 9) = **12** · ½ + ¾ × 2 = **2** · $20 - 4^2 \div 2$ = **12**. And **$(-4)^2 = 16$ but $-4^2 = -16$** — without brackets the exponent binds only to the 4.

12. Row 1: A is $1.25 each, B is $1.20 each → **B**. Row 2: A is 12 km/L, B is 12.5 km/L → **B**. Row 3: A is $2.50/kg, B is $2.60/kg → **A**. Ratios: 3 : 12 = 7 : **28** · 5 : **9** = 20 : 36 · 10 kg costs **$45** · 22 items take **55 min**.

Answers — Algebra, Data

14. 40, 34, 28, 22: initial 40, rate −6, rule 46 − 6n, term 10 = **−14**. 2.5, 4.0, 5.5: initial 2.5, rate +1.5, rule 1.5n + 1, term 10 = **16**. −8, −5, −2: initial −8, rate +3, rule 3n − 11, term 10 = **19**. 100, 87.5, 75: initial 100, rate −12.5, rule 112.5 − 12.5n, term 10 = **−12.5**. Tanks: equal after **7.5 hours** at **110 L**; Tank A empties first, at 16⅔ hours.

15. A: ½, 1¼, 2, **2¾**, **3½**; rule ¾n − ¼. B: −3, −1.5, 0, **1.5**, **3**; rule 1.5n − 4.5. C: 12, 9.5, 7, **4.5**, **2**; rule 14.5 − 2.5n. C first goes below zero at term **6** (−0.5).

16. (4a + 2) + (3a + 7) = **7a + 9** · (6n − 1) + (−2n + 9) = **4n + 8** · (5y + 3) + (−5y − 3) = **0** · (−2p + 8) + (7p − 12) = **5p − 4** · 9m − 4m = **5m** · 3k − 11k = **−8k**. The third pair are exact opposites, so everything cancels.

17. 5n − 2 → 18, 0.5, −17, −4.5. 10 − 4n → −6, 8, 22, 12. $n^2 + 1$ → 17, 1.25, **10**, 1.25. −2n + 6 → −2, 5, 12, 7. With n = −3, $n^2 = (-3)(-3)$ = **+9**, so $n^2 + 1 = 10$.

18. 3n + 7 = −8 → n = **−5** · −4n = 20 → n = **−5** · 2n − 9 = −15 → n = **−3** · 5n + 3n = −24 → n = **−3** · 1.5n + 2 = −4 → n = **−4** · −2n + 5 = 11 → n = **−3**. All six are negative here — the right-hand sides being negative, or the coefficient being negative, is the clue.

19. 3n + 4 < 19 → **n < 5**, no flip. −2n < 6 → **n > −3**, **flipped**. −5n + 3 ≥ 18 → **n ≤ −3**, **flipped**. 4n − 7 > −19 → **n > −3**, no flip. The sign flips only when you divide by a negative.

20. total = **167** · hot_days = **2** (31 and 28) · mean ≈ **23.86** · it prints **"Normal opening"**. The smallest change: raise any one temperature above 25, which makes hot_days = 3 and flips the decision.

21. One variable: class heights, most common eye colour, monthly rainfall. Two variables: height and shoe size, sleep and score, temperature and ice cream sales. The giveaway is that they all ask about a **relationship**.

23. Practice and skill: strong positive. Temperature and heating costs: strong negative. Shoe size and test score: none. Age of car and resale price: strong negative. Height and arm span: **very strong positive**. Pets and sleep: none.

24. Shoe size and reading — **age**. Firefighters and damage — **size of the fire**. Hospitals and illnesses — **population**. Study and score is the one where a real cause is plausible, and the only way to tell is to change one thing on purpose and hold the rest still.

25. Height and arm span → **scatter plot**. Four sports out of 30 → **circle graph**. Daily temperature → **broken-line graph**. Heights in bands → **histogram**. Five sports counted → **bar graph**.

26. Three coins: P(three heads) = **1/8** · P(exactly two heads) = **3/8** · P(at least one tail) = **7/8**. Three blues without replacing: 5/8 × 4/7 × 3/6 = **60/336 = 5/28**. The bottom number falls because each marble kept leaves one fewer in the bag.

Grown-ups: sheet 24 is the most useful page in the book for adult life.

Answers — Spatial Sense, Money

27. Triangle: 360 ÷ 60 = **6**, tessellates. Square: 360 ÷ 90 = **4**, tessellates. Pentagon: 360 ÷ 108 = **3.33…**, does **not**. Hexagon: 360 ÷ 120 = **3**, tessellates. Octagon: 360 ÷ 135 = **2.67**, does not. Only whole numbers work.

29. 1 : 50, 12 cm → **6 m** · 1 : 100, 6.5 cm → **6.5 m** · 1 : 25, 3 m → **12 cm** · 1 : 200, 15 m → **7.5 cm**. The room: 12 cm² on the plan; really 2 m × 1.5 m = **3 m²**. Lengths × 50, so **areas × 2 500**. Redrawing at 1 : 100 halves every measurement on the page.

30. Reflect in y-axis: A(−1,1), B(−3,1), C(−1,2). Rotate 180°: A(−1,−1), B(−3,−1), C(−1,−2). Translate 2 left 4 down: A(−1,−3), B(1,−3), C(−1,−2). Dilate ×2: A(2,2), B(6,2), C(2,4). The **dilation** is the odd one out — it changes every length and keeps every angle, so the image is *similar*, not congruent.

31. tera = 1 000 000 000 000 · giga = 1 000 000 000 · mega = 1 000 000 · micro = 0.000001 · nano = 0.000000001 · pico = 0.000000000001. 2 GB = **2×10^{9} bytes** · 500 nm = **5×10^{-7} m** · 3 MW = **3 000 000 W** · 40 µm = **0.00004 m**. There are **1 000 000** nanometres in a millimetre ($10^{-3} \div 10^{-9} = 10^{6}$).

32. Eight angles, only **two** different sizes. Corresponding to 65° is **65°**. Co-interior with 65° is **115°**. Alternate to 112° is **112°**. A pentagon's interior angles total (5 − 2) × 180 = **540°**.

33. Squares of 9, 16 and 25. And **9 + 16 = 25** — the two smaller areas add to the largest.

34. Hypotenuses: legs 6 and 8 → **10** · 5 and 12 → **13** · 9 and 12 → **15**. Legs: c = 13, b = 5 → **12** · c = 25, b = 24 → **7** · c = 10, b = 6 → **8**. The ladder reaches ≈**4.77 m** up the wall. A hypotenuse of 7 with legs of 6 and 8 is impossible because the hypotenuse must be the longest side.

35. L-shaped room: 24 − 4 = **20 m²**. Rectangle plus semicircle: 60 + 14.13 ≈ **74.13 cm²**. Cylinder on a cube: 282.6 + 512 ≈ **794.6 cm³**. Running track perimeter: two 80 m straights plus one full circle of diameter 60 m = 160 + 188.5 ≈ **348.5 m** — the two 60 m ends are *not* part of the outline.

37. Simple at 5% on $1 000: year 2 **$1 100**, year 3 **$1 150**, year 5 **$1 250**, year 10 **$1 500**. Compound: year 2 **$1 102.50**, year 3 **$1 157.63**, year 5 **$1 276.28**, year 10 **$1 628.89**. The gap at ten years is about **$128.89**.

38. $40.00 + 13% = **$45.20** · $129.99 + 13% ≈ **$146.89** · $56.50 ÷ 1.13 = **$50.00** before tax.

39. Paying in full, $500/month: Card A returns $60, Card B returns $240 − $120 = **$120**, Card C returns −$39. **Card B wins.** Carrying $2 000 all year: A costs about $400 − $60 = $340, B costs about $460 + $120 − $240 = $340, C costs about $260 + $39 = **$299**. **Card C wins.** The deciding fact is whether the balance is cleared every month.

40. $100 less 30% is $70; less a further 10% is **$63** — a real discount of **37%**, not 40%.

41. $300 CAD = **$219 USD** · €250 ≈ **$367.65 CAD** · $500 CAD = **¥55 000**. The hidden cost: at 0.73 the $100 USD purchase is **$136.99 CAD**; at the bank's 0.71 it is **$140.85**; plus 2.5% it is **$144.37**. That is **$7.38 extra, about 5.4%**.

The sheets with no answers here are not mistakes. Anything asking you to explain, design, predict or justify is judged on whether the reasoning holds together — not on matching a particular sentence.

Grown-ups: in Ontario's four achievement categories the open sheets assess **Thinking** and **Application**, which is where Level 4 lives. Accuracy alone is Level 3. **Growing Success, 2010**