Grade 7 Math
Working Below the Line

42 sheets · every strand · the year you calculate with negatives

THIS BOOK BELONGS TO

....

Find your strand

Number	a billion · square roots · rational numbers · integer arithmetic · prime factors · exponents	Sheets 1–16
Algebra	rate and initial value · subtracting monomials · sub-programs	Sheets 17–22
Data	circle graphs · what one new value does to the mean · dependent events	Sheets 23–28
Spatial Sense	dilations · angle pairs · the area of a circle · cylinders and volume	Sheets 29–38
Money	exchange rates · budgets you track · what pushes you to spend	Sheets 39–42

Brand new this year

Integer arithmetic
not just reading them

Rational numbers
one word for all of it

Square roots
and perfect squares

Order of operations
named and required

Prime factorisation
factor trees

Exponents
repeated multiplication

Fraction ÷ fraction
both of them now

Sub-programs
code you can call

Dilations
same shape, new size

Circle graphs
360° of percentages

Area of a circle
πr^2, derived

Exchange rates
money across borders

Grown-ups: two expectations carry the most weight this year. `B2.4` — arithmetic with integers, which every later grade assumes is automatic. And `B2.10` — deciding whether a situation is *proportional at all*, which is genuinely harder than solving a ratio problem and is what secondary mathematics leans on hardest.

Ontario 2020 · 52 expectations

A billion, and powers of ten

Nine zeros. And a shorter way of writing all those zeros down.

expand | compare

Expanded form using powers of ten

$4\,250\,000 = (4 \times 10^6) + (2 \times 10^5) + (5 \times 10^4)$

10^6 means six tens multiplied together — a million. The little number counts the zeros.

Power of ten	Written out	Its name
10^3		
10^6		
10^9		

Write each in expanded form.

7 300 000 = ...

1 040 000 000 = ...

How big is a billion, really? A million seconds is about 11½ days. Work out roughly how long a billion seconds is.

Answer in years: ....

A billion seconds is about 32 years. The gap between a million and a billion is far larger than the words suggest — and that gap matters every time you read a news story about money.

Grown-ups: the million/billion time comparison is the fastest way to make the scale real, and it transfers straight to reading budgets and news.

Numbers that really are squares

16 tiles make a 4 by 4 square. 15 tiles make no square at all. That is what "perfect square" means — and almost nobody is told.

build it calculate

n	1	2	3	4	5	6	7	8	9	10	11	12
n^2	1	4	9									

Square roots — the question in reverse.

$\sqrt{49}$ = ... $\sqrt{81}$ = ...

$\sqrt{144}$ = ... $\sqrt{100}$ = ...

Between two squares. $\sqrt{50}$ is not a whole number. Which two whole numbers does it sit between, and which is it closer to?

Prove it with tiles. Draw the square you can make from 25 counters. Now try 24. What goes wrong?

The word "square" is not a coincidence. Neither is "cube" — 8 counters really do stack into a 2×2×2 cube.

One word for all of them

A **rational number** is anything you can write as one integer over another. That quietly includes every whole number, every fraction, every terminating decimal — and all their negatives.

convert order

Number	As a fraction of two integers	Rational?
7	7/1	**yes**
−3		
0.25		
−2.5		
−3/4		
0		

Mark each of these: **−1.5 3/4 −2/3 2.25 −0.4**

Put in order, smallest first	Your answer
−1/2, 0.3, −0.75, 1/4, −1	
−2.5, −5/2, −2, −3, −2.05	

Look at the second one carefully. Two of those five are the same number wearing different clothes. Which two?

Squeezing in between

Name a fraction between 1/3 and 1/2. Most people freeze. There is a method, and once you have it there is no gap you cannot fill.

8 find one | then another

Two methods that always work

1. Common denominator. 1/3 = 4/12 and 1/2 = 6/12, so 5/12 sits between them.

2. Take the average. Add the two numbers and halve the result. It always lands in the middle.

Between	and	One number in between	Method used
1/4	1/3		
0.6	0.7		
–1/2	–1/3		
2/5	1/2		

Now keep going. Find a number between 0.6 and your answer. Then between 0.6 and *that*. How many times could you do this?

Forever. There is no "next" fraction and no "next" decimal — between any two numbers there is always another. That property has a name: the rational numbers are **dense**.

Grown-ups: `B1.5` looks like a small expectation and carries a large idea. Density is what separates the rationals from the integers.

Adding and subtracting below zero

The big one. Forget "two minuses make a plus" — it is a rule people misapply. Think about **distance and direction** on the line.

walk it | calculate

Two questions that look alike and are not

5 + (−3) — start at 5 and move 3 to the *left*. You land on 2.

5 − (−3) — asks *how far is it from −3 to 5?* Count the gap: 8.

Same digits. Completely different questions.

−4 + 7 = ...

3 − 8 = ...

−6 + (−5) = ...

2 − (−9) = ...

−10 − (−4) = ...

−7 + 7 = ...

Sort your six answers. Which came out positive, which negative, which zero? Then finish this:

"Subtracting a negative is the same as…"

A bank balance makes this concrete. If someone takes away a $30 debt you owed, you are $30 better off. Removing a negative leaves you higher up.

Grown-ups: the distance framing for subtraction survives contact with harder problems; the "two minuses" rule does not, because children apply it to 5 + (−3).

Integers in real situations

Every one of these is an integer problem. Write the calculation before you write the answer.

set it up | solve

Situation	The calculation	Answer
It was −8°C at dawn and rose 13 degrees by noon.		
A submarine at −120 m descends another 45 m.		
You owe $40 and pay back $25.		
A golfer is 3 under par, then goes 2 over on the next hole.		
The lift goes from floor 4 to level −2.		
Death Valley is −86 m; Mount Everest is 8848 m. What is the difference?		

The last one is a subtraction of a negative. Write it out as a calculation and explain why the answer is bigger than 8848.

Zero is not "nothing" here. It is a chosen starting point — sea level, freezing, par, no debt. Somebody decided where to put it.

Grown-ups: the Everest/Death Valley question is the best subtraction-of-a-negative problem there is, because the answer genuinely must exceed 8848.

B2.4 · B1.3

The order you do things in

BEDMAS is usually taught as a queue. It isn't one — and that is where the mistakes come from.

evaluate | check

Two ranks, not six steps

1. Brackets
2. Exponents
3. Multiplication *and* division — **same rank, left to right**
4. Addition *and* subtraction — **same rank, left to right**

The classic trap

$12 \div 3 \times 2$. Division does not wait for multiplication. Go left to right: $12 \div 3 = 4$, then $4 \times 2 = 8$. Not 2.

$20 \div 4 \times 5 =$ ....

$3 + 4 \times 2^2 =$ ....

$18 - 6 + 3 =$ ....

$(5 + 3)^2 \div 4 =$ ....

$2 \times 3^2 - 10 =$ ....

$40 \div (2 + 3) \times 2 =$ ....

Look at **18 − 6 + 3**. Somebody says the answer is 9 because addition comes before subtraction. Explain why they are wrong.

Grown-ups: 18 − 6 + 3 = 15, not 9. The "same rank, left to right" idea fixes both classic traps at once.

Breaking a number into primes

Every whole number above 1 is either prime, or a product of primes — in exactly one way. A factor tree finds it.

build a tree | write it out

A factor tree for 60

60 splits into 6 × 10. Then 6 splits into 2 × 3, and 10 into 2 × 5. Keep going until every branch ends on a prime.

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

36

84

100

Number	Prime factorisation	Using exponents
36		
84		
100		

It doesn't matter where you start. Split 60 as 4 × 15 instead of 6 × 10 and you still end up with 2 × 2 × 3 × 5. Try it and see.

Grown-ups: that the answer is the same whichever way you split is the Fundamental Theorem of Arithmetic, and Grade 7 can absolutely notice it without the name.

B2.6 · B2.7

A shorter way to write repeated multiplying

$2 \times 2 \times 2 \times 2 \times 2$ is 2^5. The small number counts **how many twos**, not what to multiply by.

convert | evaluate

Written out	In exponential notation	Value
$3 \times 3 \times 3$		
5×5		
	2^6	
	10^4	
$7 \times 7 \times 7$		

The trap. Is 2^5 the same as 2×5? Work out both and say what the small number is actually telling you to do.

2^5 = ... 2×5 = ...

The folding problem. A sheet of paper doubles in thickness each fold. How thick after 10 folds, in sheets? Write it as a power first.

As a power: .... As a number: ....

Doubling gets out of hand fast. Twenty folds would be over a million sheets thick. That is why exponents matter.

Fraction divided by fraction

Grade 6 divided whole numbers by fractions. Now both of them are fractions — and the question is still "how many fit inside?"

picture it | 8 calculate

3/4 ÷ 1/8

How many eighths fit into three quarters? Three quarters is six eighths, so **6**.

The shortcut: flip the second fraction and multiply. 3/4 × 8/1 = 24/4 = 6. Same answer.

1/2 ÷ 1/4 = ...

3/4 ÷ 1/2 = ...

5/6 ÷ 1/3 = ...

2/3 × 3/4 = ...

1/2 × 2/5 = ...

3/5 × 5/9 = ...

Before you use the shortcut, predict. Will 5/6 ÷ 1/3 be bigger or smaller than 5/6? Say why, then check.

Adding and subtracting — make equivalent fractions first.

2/3 + 1/4 = ...

3/5 + 1/2 = ...

7/8 − 1/6 = ...

5/6 − 3/4 = ...

Rewrite both over a common denominator, then add or subtract the tops. The bottoms never get added.

Dividing by something less than one always makes it bigger. Multiplying by something less than one always makes it smaller. Check every answer above against that.

Decimal times decimal

Multiply as if the points weren't there, then count how many decimal places went in — that's how many come out.

estimate calculate

☑ **0.4 × 0.3**

Ignore the points: 4 × 3 = 12. Two decimal places went in, so two come out: **0.12**.

Sanity check: less than half of less than a half. It should be small.

Question	Estimate	Exactly
0.6 × 0.5		
1.2 × 0.4		
2.5 × 1.5		
4.8 ÷ 0.6		
0.96 ÷ 0.4		

0.6 × 0.5 = 0.3. A classmate says multiplying should make it bigger, so this must be wrong. What would you say to them?

For division, make the divisor whole. 4.8 ÷ 0.6 becomes 48 ÷ 6 — multiply both by 10 and the answer doesn't change.

Grown-ups: the place-counting rule works, but only sticks if it is checked against an estimate each time.

Going up and coming down

Grade 6 found percents *of* things. Grade 7 uses them to **change** things — and the direction matters more than you'd think.

mental math | compare

Start	+10%	−10%	+25%	−25%	+100%
80					
200					
$60					

The quick way

To add 10%, multiply by **1.1**. To take 10% off, multiply by **0.9**. To add 25%, multiply by **1.25**. To take 25% off, multiply by **0.75**. Doubling is × 2, or +100%.

The one that catches everybody. A $100 item goes up 10%, then down 10%. Is it back to $100?

After the rise: .... After the fall: ....

Explain why, using the two starting numbers.

No — you get $99. The 10% rise was of $100; the 10% fall was of $110. Percents are always *of* something, and the something changed.

Is it even proportional?

The most important sheet in the book. Once you learn to multiply across, you will want to do it to everything — and half the time you shouldn't.

decide first | then solve

Situation	Proportional?	Why or why not
3 pens cost $4.50, so 6 pens cost…		
A 3-year-old is 90 cm tall, so a 6-year-old is…		
1 person paints a fence in 6 hours, so 2 people take…		
A car does 240 km on 20 L, so on 40 L it does…		
It takes 9 months to grow a baby, so 9 people take…		
4 tickets cost $52, so 7 tickets cost…		

The test

A situation is **proportional** if doubling one quantity doubles the other, and if **zero goes with zero**. Zero pens cost zero dollars — proportional. A zero-year-old is not zero centimetres tall — not proportional.

Now solve only the proportional ones. Show the unit rate you used.

The painting one is interesting. Two people take *less* time, not more — the quantities move in opposite directions. That is called inverse, and it is not proportional in the ordinary sense.

Simplifying, and switching form

A fraction in lowest terms is easier to think about. And every fraction has a decimal and a percent twin.

simplify · convert

Simplifying 42/56

The greatest common factor of 42 and 56 is 14. Divide both by 14: 3/4.

A factor tree gets you there if you cannot see the GCF straight away.

18/24 = ... **45/60 =** ....

36/48 = ... **63/81 =** ....

Fill every gap.

Fraction (lowest terms)	Decimal	Percent
3/8		
	0.45	
		150%
5/6		

Know these without working them out

1/2 = 0.5 = 50% · 1/4 = 0.25 = 25% · 3/4 = 0.75 = 75% · 1/3 ≈ 0.333 ≈ 33.3%
1/5 = 0.2 = 20% · 1/8 = 0.125 = 12.5% · 1/10 = 0.1 = 10% · 2/3 ≈ 0.667 ≈ 66.7%

Cover the box above and fill these in from memory.

3/4 = ... **=** .... **1/8 =** .... **=** ....

1/3 ≈ .... **≈** .... **1/5 =** .... **=** ....

150% is more than a whole. As a fraction that is 3/2, or one and a half. Percents are not capped at 100.

Grown-ups: `B2.2` asks for *recall*, not calculation — these eight are worth knowing cold. 5/6 gives 0.8333..., a good moment to revisit rounding.

B1.4 · B1.7 · B1.6 · B2.2

Number — how am I doing?

Tick honestly. A blank is information, not a failure.

☑ tick

- ☐ I can write a big number using powers of ten
- ☐ I can find a square root
- ☐ I can find a number between any two
- ☐ I know why 5 − (−3) = 8
- ☐ I can build a factor tree
- ☐ I can divide a fraction by a fraction
- ☐ I can increase and decrease by a percent
- ☐ I know what a perfect square is, and why
- ☐ I know what "rational number" covers
- ☐ I can add and subtract integers
- ☐ I know multiplication and division are the same rank
- ☐ I can use exponential notation
- ☐ I can multiply two decimals
- ☐ I can tell whether a situation is proportional

The one I want to get better at is…

…and the sheet that would help is number ....

Grown-ups: the integer rows and the proportional row are the two to protect. Everything in Grade 8 assumes both.

Two patterns, side by side

Grade 6 could say a pattern was linear. Grade 7 puts two next to each other and asks which grows faster, and which started higher.

8 compare | graph

Two numbers describe any linear pattern

The **initial value** — where it starts, before anything happens. The **constant rate** — how much it changes each step. That's it. Those two numbers fix the whole pattern.

Pattern	Initial value	Constant rate	Rule
5, 8, 11, 14, ...			
20, 18, 16, 14, ...			
1.5, 3.0, 4.5, 6.0, ...			

Two phone plans. Plan A: $20 up front plus $5 a month. Plan B: $35 up front plus $2 a month.

Months	1	2	3	4	5	6
Plan A						
Plan B						

Which is cheaper at 2 months? .... At 6 months? ....

They cross over somewhere. When? ....

"Which is cheaper" has no single answer. It depends how long you keep it — and that is exactly what comparing rate and initial value tells you.

Grown-ups: initial value and constant rate are y-intercept and slope, a year before either word appears. The crossover question is `C1.1` doing real work.

C1.1 · C1.2

Rules, forwards and backwards

Write the rule for term *n*. Then use it to jump straight to term 50 — and to work backwards from a value.

rule solve

Term (n)	Value
1	7
2	12
3	
4	
20	
n	

Initial value ...., constant rate ....

Rule: value = ...n + ...

Backwards. Which term has the value 97? Solve it as an equation, don't list.

Now one with decimals.

Term	1	2	3	4	10	n
Value	0.5	1.7				

A pattern with a decimal rate behaves exactly the same way. Only the arithmetic changes.

Patterns that go below zero

Same idea, negative territory. This is where the integer work from sheet 5 meets algebra.

extend · describe

Pattern	Constant rate	Next three	Rule
10, 6, 2, −2, ...			
−12, −9, −6, −3, ...			
5, 0, −5, −10, ...			
−1, −3, −5, −7, ...			

Which of these ever reaches zero? Say how you can tell without listing them all out.

Make your own. Write a pattern that starts positive, crosses zero, and has a constant rate of −7.

A shrinking pattern is still linear. The rate is just negative. Nothing else about it changes.

Grown-ups: patterns crossing zero are where sign errors show up, and they are worth doing slowly.

Taking terms away

Grade 6 only added monomials. $7n - 3n$ is easy. **$3n - 7n$** is where almost everybody trips.

8 collect | ☑ check signs

☑ Why $3n - 7n = -4n$

Three lots of n, take away seven lots of n. You cannot take seven from three without going below zero — so you end up **four lots of n in the negative.**

It is exactly $3 - 7 = -4$ from sheet 5, with an n attached.

$9x - 4x =$ ....

$2a - 8a =$ ....

$5m - 5m =$ ....

$-3y + 7y =$ ....

$6n - 2n - 9n =$ ....

$4p + 3p - 10p =$ ....

Now put values in.

Expression	n = 3	n = 0.5	n = −2
$4n - 9$			
$2n + 5n$			
$10 - 3n$			

The last column is the hard one. $10 - 3n$ with $n = -2$ means $10 - 3 \times (-2)$. Take your time with the signs.

Equations and inequalities, with decimals

> Undo in reverse. Verify every time — it is written into the expectation, not an optional extra.

8 solve | verify

Equation	n =	Check — write it out
$4n + 7 = 35$		
$2.5n = 15$		
$3n - 1.5 = 10.5$		
$5n + 2n = 42$		
$0.5n + 4 = 9$		

Inequalities. Shade every whole number that works.

$3n + 4 < 25$

$2n - 3 > 11$

Test your shading. Pick one number you shaded and one you didn't. Substitute both. Does each behave as it should?

Grown-ups: both `C2.3` and `C2.4` say *verify*. A solution without a substitution back has met half the expectation.

Code you can call by name

A **sub-program** is a chunk of code with a name. Write it once, use it anywhere. It is the most important idea in programming.

define | trace

From a loop to a named block

GRADE 6 — a loop

```
repeat 4 times:
  forward 10
  turn right 90
```

GRADE 7 — a sub-program

```
define square:
  repeat 4 times:
    forward 10
    turn right 90

repeat 4 times:
  square
  forward 10
```

The second one draws a row of four squares — and you only described a square once.

Write a **sub-program** called **triangle** that draws an equilateral triangle. How far does it turn each time?

Turn angle: .... hint: the turns must add to 360°

Why does this matter? Suppose you want the squares to be size 20 instead of 10. How many places would you change in each version?

One place instead of four. That is the whole argument for sub-programs, and it only gets stronger as programs grow.

Why big numbers become percentages

"4 200 000 people" tells you almost nothing on its own. "31% of the population" tells you a great deal.

convert explain

Population	Number affected	As a percentage
2 000	500	
40 000	10 000	
1 200 000	300 000	

All three are the same percentage. Which fact is more useful when you are comparing three towns — the count, or the percentage? Say why.

Now the other way round. Town A: 8% of 50 000. Town B: 12% of 20 000. Which town has *more people* affected?

A: .... B: .... So: ....

Percentages and counts answer different questions. Town B has the higher rate; Town A has more people. Both are true, and a news story can choose whichever suits it.

Grown-ups: `D1.1` asks students to explain *why* percentages are used. The last box shows the flip side — a percentage can hide the size of a problem. D1.1 · D1.2

Circle graphs — 360 degrees of percent

Every slice is a share of the whole. To draw one you need percents, and then you need to turn those into angles.

8 calculate | construct

From a count to a slice

12 of 40 students walk. That is 12 ÷ 40 = 0.3 = **30%**. The slice is 30% of 360°, which is **108°**.

How they travel	Students	%	Angle
Walk	**12**		
Bus	**16**		
Car	**8**		
Bike	**4**		
Total	**40**		

Draw the graph in the circle. Use a protractor.

Two checks before you stop. Do your percentages add to 100? Do your angles add to 360?

% total: .... Angle total: ....

A circle graph can't show change over time, and it can't compare two populations of different sizes. It shows how one whole divides up — nothing else.

Grown-ups: the two totals are the built-in error check, and insisting on them catches nearly every mistake.

D1.3 · D1.4

What one new number does

Add a single very large value to a data set and watch what happens. The mean moves. The median barely notices.

8 calculate | compare

Seven weekly wages at a small shop, in dollars:

400, 420, 450, 400, 470, 430, 430

As it stands

mean ....

median ....

range ....

Now the owner adds their own: $3 500

mean ....

median ....

range ....

Which measure moved most? And which one better describes what a typical worker earns?

So why does the news usually say *median* income?

An extreme value is called an outlier. The mean feels every outlier; the median mostly shrugs. Now you know which one to be suspicious of.

Graphs that mislead without lying

Every trick here uses true numbers. That is precisely why you have to look at the axis yourself.

inspect redraw

Four ways it is done:

1. The axis starts partway up, so small gaps look enormous
2. The scale is uneven, so equal steps aren't equal
3. Only the convenient years are shown
4. A circle graph whose slices don't add to 100%

Axis from 90 — plot 94, 95, 96, 97.

Axis from 0 — the same four numbers.

Is either graph false? Both plot the same four numbers accurately. So what exactly has changed?

Neither is false. Both are honest about the numbers and dishonest about the impression. That is why "check the axis" is the single most useful habit in reading data.

Grown-ups: the answer to "is either false?" is no, and getting a student to say so out loud is worth more than a list of tricks.

Did you put it back?

That is the whole difference between independent and dependent events. One question, and it changes everything.

decide calculate

Situation	Independent or dependent?
Flip a coin twice	
Draw a card, put it back, draw again	
Draw a card, keep it, draw again	
Take a marble from a bag and eat it, then take another	
Roll two dice at once	
Pick two names from a hat without replacing	

A bag with 3 red and 5 blue marbles

With replacement: P(red) = 3/8 both times. The bag never changes.

Without: P(red) = 3/8 first, then 2/7 — one red gone, and one marble fewer altogether.

Your turn. Same bag. What is the chance of drawing two reds in a row, *without* replacing?

First draw .... × second draw .... = ...

And *with* replacing? ....

Try it thirty times each way and pool the class results. Do the two experimental figures separate the way the theory says they should?

Grown-ups: "did you put it back?" is the entire distinction, and a bag of marbles settles it faster than any definition.

D2.1 · D2.2

Sorting the solids

Prisms, pyramids and cylinders. And two kinds of symmetry that most people have never separated.

classify

☑ compare

What makes each one

A **prism** has two identical parallel bases joined by rectangles. Its name comes from the base. A

pyramid has one base and triangular faces meeting at a point. A

cylinder is a prism whose base is a circle.

Solid	Faces	Edges	Vertices	Shape of base
Triangular prism				
Rectangular prism				
Square-based pyramid				
Hexagonal prism				

Plane symmetry

a flat cut giving two mirror halves

How many planes of symmetry has a cube? ....

Rotational symmetry

turn it and it looks unchanged

Spin a cylinder about its long axis. How many positions look identical? ....

A cylinder has infinite rotational symmetry about its axis — every angle looks the same. Very few shapes can say that.

Grown-ups: plane symmetry is a cut; rotational symmetry is a turn. Keeping them separate is what `E1.1` is asking for.

Same shape, different size

A **dilation** is the new transformation. Slides, flips and turns keep the size. A dilation changes it — and keeps the shape.

enlarge | 8 scale

Congruent or similar?

Congruent — identical in shape *and* size. Translations, reflections and rotations all give congruent images.

Similar — same shape, different size. That is what a dilation gives you.

Dilate by a scale factor of 2. Draw a small triangle, then double every side.

Measure both.

	Original	Image
Side a		
Side b		
Angle A		

Sides: .... Angles: ....

The key finding. The sides doubled. What happened to the angles? And what does that tell you about what a dilation preserves?

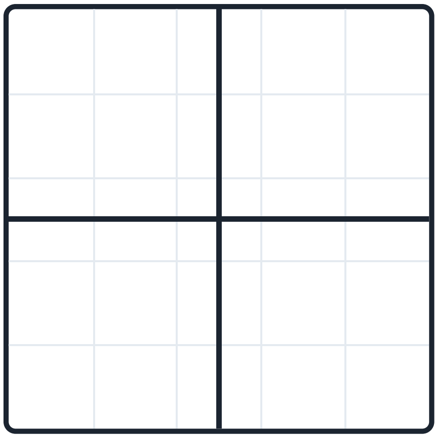
Scale factor less than 1 shrinks it. A factor of 0.5 halves every side — and it is still a dilation, and still similar.

Grown-ups: that angles survive a dilation while lengths do not is the definition of similarity, and every map and scale drawing depends on it.

Transformations by coordinate

On a grid you flipped shapes by eye. On the Cartesian plane you can **predict the numbers** — which is where geometry starts turning into algebra.

plot | 8 predict



Label both axes −5 to +5. Plot triangle P(2,1), Q(4,1), R(2,4).

Transformation	P becomes	Q becomes	R becomes
Reflect in the y-axis			
Reflect in the x-axis			
Rotate 180° about (0,0)			
Translate 3 left, 2 down			

Find the rules. Complete each one:

Reflect in the y-axis: (x, y) → ....

Reflect in the x-axis: (x, y) → ....

Rotate 180° about the origin: (x, y) → ....

You just wrote geometry as algebra. Those three rules work for every point on the plane — you never have to draw it again.

Drawing what you see, to scale

Top, front and side — plus a perspective view. And every one of them at a stated scale.

draw   scale

Choose an object or a room. What I am drawing: ....

My scale: 1 cm represents ....

top	front	side

Now a perspective view — how it actually looks to your eye.

Which view is most useful to a builder, and which to a buyer? Say why they are different.

Grown-ups: the builder wants measurable orthographic views; the buyer wants the perspective. That distinction is the point of `E1.2`.

Angles that come in pairs

Four relationships. Once you know them you can find missing angles without measuring anything.

calculate | justify

The four you need

Complementary	— two angles adding to	**90°**	**Supplementary**	— two angles adding to	**180°**	**Opposite**	(vertically opposite) — where two lines cross, the angles facing each other are	**equal**	**Interior angles of a triangle**	add to	**180°**	; of a quadrilateral,	**360°**	to

Given	Find	Which rule did you use?
One angle is 35°, its complement is…		
One angle is 118°, its supplement is…		
Two lines cross; one angle is 72°. The opposite one is…		
A triangle has angles 47° and 63°. The third is…		
A quadrilateral has 90°, 85°, 110°. The fourth is…		

An exterior angle. Extend one side of a triangle. The angle outside is supplementary to the interior one next to it. If the interior angle is 65°, what is the exterior angle?

Every answer here should come with a reason. "It's 145° because angles on a straight line add to 180°" is a complete answer. Just the number isn't.

Grown-ups: `E2.3` is about *solving for unknowns using properties* — the justification is the assessed part, not the arithmetic. E2.3

Where πr^2 comes from

You are not being given this formula. You are going to cut a circle up and watch it appear.

cut · rearrange

The wedge trick

Cut a paper circle into 16 thin wedges. Lay them alternately point-up and point-down, side by side. You get something very close to a **rectangle**.

That rectangle is r high — each wedge is a radius long.
And it is **half the circumference** long, because half the wedges point up and half point down.

half of $2\pi r = \pi r$ · so area $= \pi r \times r = \pi r^2$

Do it. Cut and rearrange, then sketch what you got.

Now use it ($\pi \approx 3.14$)

r = 5 cm → area ....

r = 10 cm → area ....

diameter 8 cm → area ....

Double the radius. Compare the areas for r = 5 and r = 10. Did the area double?

Why not?

Doubling the radius quadruples the area — because the radius appears twice in the formula. A 12-inch pizza is not twice a 6-inch pizza. It is four times.

Grown-ups: `E2.5` says *develop the formula*. The wedge construction takes ten minutes and is the difference between understanding πr^2 and memorising it. E2.5

Cut it into pieces you know

Trapezoids, rhombuses, kites and awkward composite shapes. Stop hunting for a formula — draw the cutting line.

decompose | 8 add up

A trapezoid

Cut it into a rectangle and two triangles. Work out each area, add them. You never needed the trapezoid formula.

Draw your cutting lines on each shape, then label the pieces.

Show your working

Piece	Its area
Total	

The pieces you already know

rectangle = length × width · triangle = base × height ÷ 2
parallelogram = base × height · circle = πr^2

A rhombus, two ways. Cut it into four triangles. Or use the diagonals: area = $(d_1 \times d_2) \div 2$. Try both on the same shape and check they agree.

Grown-ups: insisting the cutting line is drawn on the diagram is the whole skill. A student who tries to do it in their head will lose track.

One rule for every prism and cylinder

Work out the area of the base. Multiply by how tall it is. That is the whole thing — for all of them.

8 calculate | generalise

Why it works

A prism is the same shape stacked over and over. One layer holds (area of base) cubes. Stack it *h* layers high and you have base × height.

Volume = area of base × height

Solid	Area of base	Height	Volume
Rectangular prism, base 6 × 4 cm		**10 cm**	
Triangular prism, base 8 cm wide × 5 cm high		**12 cm**	
Cylinder, radius 3 cm		**10 cm**	
Cylinder, radius 5 cm		**4 cm**	

Work backwards. A cylinder has a volume of 502.4 cm^3 and a height of 8 cm. What is the area of its base? And its radius?

Base area .... Radius ....

A cylinder is just a prism with a circular base. Once you see that, you need one volume formula instead of five.

Grown-ups: `E2.7` explicitly asks for finding any one of base area, volume or height from the other two — the backwards question is part of the expectation. E2.7

Unrolling a cylinder

Cut the label off a tin and lay it flat. It is a rectangle — and one of its sides is the circumference.

unroll | 8 add

The net of a cylinder

Two circles — the top and bottom, each πr^2.
One rectangle — the curved side unrolled. It is **$2\pi r$** long (the circumference) and **h** high.

Surface area = $2\pi r^2 + 2\pi rh$

Draw the net of a tin with radius 3 cm and height 10 cm. Label every part.

Work it out ($\pi \approx 3.14$)

Two circles: ....

The rectangle: ....

Total surface area: ....

Now a tin r = 5 cm, h = 4 cm: ....

Go and check. Find a real tin. Measure it, calculate its surface area, then peel the label and measure the label directly. How close were you?

The label really is a rectangle. That is the single fact that makes cylinder surface area obvious instead of memorised.

Volume, capacity, and one lucky fact

1 mL = 1 cm^3, exactly. Not a coincidence — the metric system was designed that way.

convert | distinguish

Volume

How much **space** something takes up. Measured in cm^3, m^3. Every object has one, solid or not.

Capacity

How much a container will **hold**. Measured in mL, L. Only containers have one.

From	To	From	To
250 cm^3		**1.5 L**	
2 000 cm^3		**750 mL**	
1 m^3		**3 L**	

A box 20 cm × 10 cm × 5 cm. What is its volume in cm^3? How many litres of water would it hold?

Volume .... Capacity ....

A solid brick has a volume. Does it have a capacity? Explain.

Careful: 1 m^3 is 1 000 000 cm^3, not 100. You are converting in three directions at once — the same trap as m^2 and cm^2, one dimension worse.

Spatial Sense — how am I doing?

Tick honestly. A blank is information.

tick

- [ ] I can classify prisms, pyramids and cylinders
- [ ] I can perform a dilation
- [ ] I can write a transformation as a coordinate rule
- [ ] I can use complementary and supplementary angles
- [ ] I can explain where πr^2 comes from
- [ ] I know volume = base area × height
- [ ] I know 1 mL = 1 cm^3
- [ ] I know plane symmetry from rotational symmetry
- [ ] I know the difference between congruent and similar
- [ ] I can draw views to a stated scale
- [ ] I can justify an angle answer, not just state it
- [ ] I can find the area of a composite shape
- [ ] I can find the surface area of a cylinder

The one I want to get better at is…

…and the sheet that would help is number ....

Grown-ups: "I can explain where πr^2 comes from" is the row that separates understanding from recall.

Money across a border

An exchange rate is a ratio. Everything you learned on sheet 13 applies here.

8 convert compare

Reading a rate

If **$1 CAD = $0.73 USD**, then to go from Canadian to American you **multiply** by 0.73. To come back the other way you **divide**.

Always ask first: should this answer be bigger or smaller?

Amount	Rate	Converted	Bigger or smaller?
$100 CAD	**1 CAD = 0.73 USD**		
$50 USD	**1 CAD = 0.73 USD**		
$200 CAD	**1 CAD = 0.68 EUR**		
¥5000 JPY	**1 CAD = 110 JPY**		

Go and look one up. Find today's rate between the Canadian dollar and one other currency. Write it down with the date.

1 CAD = ... on ....

Check it again in a month. Did it move? By how much as a percentage?

A rate that moves is why the same trip costs different amounts in different years. Nothing about the trip changed.

Grown-ups: exchange rates are `B2.10` proportional reasoning wearing a different hat, and they are checkable in real time. F1.1

A budget you actually keep

Grade 6 set a goal. Grade 7 has to **create, track and adjust** — and adjusting is the part nobody does.

plan · track

My longer-term goal: .... Cost: ....

Month	Money in	Money out	Saved this month	Running total
1				
2				
3				
4				

Am I on track? At this rate, when will I get there?

Originally I predicted ....

Adjust. If you're behind, name *one* specific change. Not "spend less" — an actual thing.

Where to check a number

A bank's own published rates · the Bank of Canada · the Financial Consumer Agency of Canada · a government site.

Not an advertisement, an influencer, or anyone who profits from your decision.

Grown-ups: the "one specific change" box is what makes `F1.3` different from Grade 6's goal-setting. Vague answers here mean the expectation hasn't landed.

F1.2 · F1.3

Interest over time, and who is nudging you

Two things: how interest compounds when you leave it alone, and who is trying to make you spend.

calculate · notice

$1 000 saved at 5% a year, left completely alone.

Year	Starting amount	Interest earned	New total
1	$1 000		
2			
3			
4			

Notice the interest column grows. Why does year 4 earn more than year 1, when the rate never changed?

Now the other side — what is pushing you to spend?

The nudge	How it works on you
"Only 3 left in stock!"	
A saved card so buying takes one tap	
Everyone in your group has one	
"Buy now, pay in 4 instalments"	

You can name every one of those. A person who can name the mechanism being used on them is much harder to move than one who has simply been told to be sensible.

Grown-ups: `F1.4` names *societal* factors explicitly — advertising and social pressure are on-curriculum, not a digression.

Shopping for an account

Same money, different institutions, different outcomes. This is a real skill and almost nobody teaches it.

compare | decide

Account	Interest rate	Monthly fee	After a year on $2 000
Bank A savings	**1.5%**	**$4**	
Bank B savings	**2.5%**	**$10**	
Credit union	**2.0%**	**$0**	

Which is best for $2 000? Show the working — interest earned, minus fees paid.

Now for $200 instead. Does your answer change? Why does the amount matter?

The highest rate is not always the best account. On a small balance a monthly fee can swallow the interest entirely — and that is exactly the sort of thing an advertisement will not mention.

Grown-ups: the small-balance question is the one that matters. On $200, Bank B loses money. `F1.6` is about doing that arithmetic rather than reading the headline rate.

F1.6 · F1.5

Answers — Number

> Only questions with one right answer are here. Anything asking you to explain or decide is judged on the reasoning.

1. 10^3 = 1000 (a thousand) · 10^6 = 1 000 000 (a million) · 10^9 = 1 000 000 000 (a billion). 7 300 000 = (7 × 10^6) + (3 × 10^5). 1 040 000 000 = (1 × 10^9) + (4 × 10^7). A billion seconds is about **32 years**.

2. Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144. √49 = 7 · √81 = 9 · √144 = 12 · √100 = 10. √50 sits between **7 and 8**, much closer to 7 (since 7^2 = 49). 24 counters make no square — you can get 4 × 6, but not a square.

3. −3 = −3/1 · 0.25 = 1/4 · −2.5 = −5/2 · −3/4 is already there · 0 = 0/1. All rational. Order: −1, −0.75, −1/2, 0.3, 1/4 — careful, 1/4 = 0.25 so it goes before 0.3: **−1, −0.75, −1/2, 1/4, 0.3**. Second set: −3, −2.5, −5/2, −2.05, −2 — and **−2.5 and −5/2 are the same number**.

4. Between 1/4 and 1/3 → 7/24 (twelfths won't split them; use 24ths). Between 0.6 and 0.7 → 0.65. Between −1/2 and −1/3 → −5/12. Between 2/5 and 1/2 → 9/20. You can keep going **forever** — the rationals are dense.

5. −4 + 7 = 3 · 3 − 8 = −5 · −6 + (−5) = −11 · 2 − (−9) = 11 · −10 − (−4) = −6 · −7 + 7 = 0. Subtracting a negative is the same as **adding the positive**.

6. −8 + 13 = **5°C** · −120 − 45 = **−165 m** · −40 + 25 = **−$15** (still owing $15) · −3 + 2 = **−1** (one under par) · from 4 to −2 is **6 floors down** · 8848 − (−86) = **8934 m**, bigger than 8848 because the two points are on opposite sides of zero.

7. 20 ÷ 4 × 5 = **25** · 3 + 4 × 2^2 = 3 + 16 = **19** · 18 − 6 + 3 = **15** · $(5 + 3)^2$ ÷ 4 = 64 ÷ 4 = **16** · 2 × 3^2 − 10 = 18 − 10 = **8** · 40 ÷ (2 + 3) × 2 = 8 × 2 = **16**. 18 − 6 + 3 is 15, not 9 — addition and subtraction are the same rank, so you go left to right.

8. 36 = $2^2 \times 3^2$ · 84 = 2^2 × 3 × 7 · 100 = $2^2 \times 5^2$. Splitting 60 a different way gives the same primes every time.

9. 3^3 = 27 · 5^2 = 25 · 2^6 = 64 · 10^4 = 10 000 · 7^3 = 343. 2^5 = **32** but 2 × 5 = **10** — the small number counts how many 2s, it is not a multiplier. Ten folds = 2^{10} = **1024** sheets.

10. 1/2 ÷ 1/4 = 2 · 2/3 × 3/4 = 1/2 · 3/4 ÷ 1/2 = 3/2 · 1/2 × 2/5 = 1/5 · 5/6 ÷ 1/3 = 5/2 · 3/5 × 5/9 = 1/3. 5/6 ÷ 1/3 is bigger than 5/6, because you are dividing by less than one. Adding and subtracting: 2/3 + 1/4 = **11/12** · 7/8 − 1/6 = **17/24** · 3/5 + 1/2 = **11/10** · 5/6 − 3/4 = **1/12**.

11. 0.6 × 0.5 = 0.3 · 1.2 × 0.4 = 0.48 · 2.5 × 1.5 = 3.75 · 4.8 ÷ 0.6 = 8 · 0.96 ÷ 0.4 = 2.4. Multiplying by less than one makes the answer smaller — that is not a mistake.

12. Of 80: 88, 72, 100, 60, 160. Of 200: 220, 180, 250, 150, 400. Of $60: $66, $54, $75, $45, $120. $100 up 10% is $110; down 10% of *$110* is $11, giving **$99**. Not $100 — the base changed.

13. Proportional: pens (6 cost $9.00), the car (480 km), tickets (7 cost $91). **Not** proportional: height with age, the painting job (two people take *less* time), and the nine-months one. The test is whether zero goes with zero.

14. 18/24 = 3/4 · 45/60 = 3/4 · 36/48 = 3/4 · 63/81 = 7/9. 3/8 = 0.375 = 37.5% · 0.45 = 9/20 = 45% · 150% = 1.5 = 3/2 · 5/6 = 0.8333… = 83.3%.

Grown-ups: sheet 13's "does zero go with zero?" is the most transferable idea in the strand.

Answers — Algebra, Data

16. 5, 8, 11, 14: initial value 5, rate +3, rule $3n + 2$. 20, 18, 16, 14: initial 20, rate −2, rule $22 - 2n$. 1.5, 3.0, 4.5: initial 1.5, rate +1.5, rule $1.5n$. Phone plans — A: 25, 30, 35, 40, 45, 50. B: 37, 39, 41, 43, 45, 47. A is cheaper at 2 months; B is cheaper at 6. They are **equal at 5 months** ($45), and B wins after that.

17. 7, 12, 17, 22 … initial value 7, rate +5, rule **$5n + 2$**. Term 20 = 102. Value 97 means $5n + 2 = 97$, so $5n = 95$ and **$n = 19$**. The decimal pattern: 0.5, 1.7, 2.9, 4.1, term 10 = 11.3, rule $1.2n - 0.7$.

18. 10, 6, 2, −2 → rate −4 → −6, −10, −14; rule $14 - 4n$. −12, −9, −6, −3 → rate +3 → 0, 3, 6; rule $3n - 15$. 5, 0, −5, −10 → rate −5 → −15, −20, −25; rule $10 - 5n$. −1, −3, −5, −7 → rate −2 → −9, −11, −13; rule $1 - 2n$. The second and third reach zero exactly; the others step over it.

19. $9x - 4x = 5x$ · $2a - 8a = -6a$ · $5m - 5m = 0$ · $-3y + 7y = 4y$ · $6n - 2n - 9n = -5n$ · $4p + 3p - 10p = -3p$. Evaluating: $4n - 9$ → 3, −7, −17 · $2n + 5n$ ($= 7n$) → 21, 3.5, −14 · $10 - 3n$ → 1, 8.5, **16**.

20. $n = 7$ · $n = 6$ · $n = 4$ · $n = 6$ · $n = 10$. Inequalities: $3n + 4 < 25$ gives $n < 7$, so 0 to 6. $2n - 3 > 11$ gives $n > 7$, so 8 upwards.

21. An equilateral triangle turns **120°** at each corner — the three turns must total 360°. To resize the squares you change **one** place in the sub-program version, and **four** in the copied version.

22. All three are **25%**. The percentage is more useful for comparing towns of different sizes. Town A: 8% of 50 000 = **4 000**. Town B: 12% of 20 000 = **2 400**. So Town A has more people affected, even though Town B has the higher rate.

23. Walk 12/40 = 30% = 108° · Bus 16/40 = 40% = 144° · Car 8/40 = 20% = 72° · Bike 4/40 = 10% = 36°. Totals: 100% and 360°.

24. Original seven: mean **$428.57** (to the nearest cent), median **$430**, range **$70**. With $3 500 added: mean **$812.50**, median **$430**, range **$3 100**. The mean nearly doubled; the median did not move at all. The **median** better describes a typical worker — which is exactly why income is usually reported as a median.

25. Neither graph is false. Both plot 94, 95, 96, 97 accurately. Only the *impression* changed — which is why checking the axis is the habit that matters.

26. Independent: two coin flips, draw-replace-draw, two dice. Dependent: draw and keep, eat the marble, two names without replacing. Two reds without replacing: $3/8 \times 2/7$ = **$6/56 = 3/28$**. With replacing: $3/8 \times 3/8$ = **$9/64$**.

Grown-ups: sheet 24 is the most useful page in the Data strand for adult life.

Answers — Spatial Sense, Money

27. Triangular prism: 5 faces, 9 edges, 6 vertices, triangular base. Rectangular prism: 6, 12, 8, rectangle. Square-based pyramid: 5, 8, 5, square. Hexagonal prism: 8, 18, 12, hexagon. A cube has **9** planes of symmetry. A cylinder has **infinite** rotational symmetry about its long axis.

28. After a dilation by 2, every side doubles and every **angle stays exactly the same**. That is what makes the image *similar* rather than congruent. A scale factor below 1 shrinks it, and it is still a dilation.

29. Reflect in the y-axis: P(−2,1), Q(−4,1), R(−2,4). In the x-axis: P(2,−1), Q(4,−1), R(2,−4). Rotate 180°: P(−2,−1), Q(−4,−1), R(−2,−4). Translate 3 left 2 down: P(−1,−1), Q(1,−1), R(−1,2). Rules: y-axis **(x, y) → (−x, y)** · x-axis **(x, y) → (x, −y)** · 180° **(x, y) → (−x, −y)**.

31. Complement of 35° is **55°**. Supplement of 118° is **62°**. Opposite of 72° is **72°**. Third angle of the triangle: 180 − 47 − 63 = **70°**. Fourth angle of the quadrilateral: 360 − 285 = **75°**. An exterior angle beside a 65° interior angle is **115°**.

32. r = 5 → area ≈ **78.5 cm^2** · r = 10 → ≈ **314 cm^2** · diameter 8 (so r = 4) → ≈ **50.24 cm^2**. Doubling the radius **quadruples** the area, because r appears twice in πr^2. A 12-inch pizza is four times a 6-inch one.

34. Rectangular prism: base 24 cm^2, volume **240 cm^3**. Triangular prism: base 20 cm^2, volume **240 cm^3**. Cylinder r = 3: base ≈ 28.26 cm^2, volume ≈ **282.6 cm^3**. Cylinder r = 5: base ≈ 78.5 cm^2, volume ≈ **314 cm^3**. Working backwards: 502.4 ÷ 8 = base area **62.8 cm^2**, so r^2 = 20 and the radius is about **4.5 cm**.

35. Tin r = 3, h = 10: two circles ≈ 56.52 cm^2, rectangle = 2π(3)(10) ≈ 188.4 cm^2, total ≈ **244.92 cm^2**. Tin r = 5, h = 4: 2π(25) + 2π(5)(4) ≈ 157 + 125.6 ≈ **282.6 cm^2**.

36. 250 cm^3 = 250 mL · 2 000 cm^3 = 2 L · 1 m^3 = **1 000 000 cm^3** = 1000 L · 1.5 L = 1500 cm^3 · 750 mL = 750 cm^3 · 3 L = 3000 cm^3. The box: volume **1 000 cm^3**, capacity **1 L**. A solid brick has a volume but **no capacity** — it holds nothing.

38. $100 CAD = **$73 USD** (smaller) · $50 USD ≈ **$68.49 CAD** (bigger) · $200 CAD = **€136** · ¥5000 ≈ **$45.45 CAD**.

40. Year 1: $50 interest, total $1 050. Year 2: $52.50, total $1 102.50. Year 3: $55.13, total $1 157.63. Year 4: $57.88, total $1 215.51. The interest grows because each year it is calculated on a larger amount — that is **compounding**.

41. On $2 000 — Bank A: $30 interest − $48 fees = **−$18**. Bank B: $50 − $120 = **−$70**. Credit union: $40 − $0 = **+$40**. On $200 — A: $3 − $48 = −$45. B: $5 − $120 = −$115. Credit union: $4 − $0 = +$4. The credit union wins both times, and the gap is worse on the smaller balance. **The highest rate is not the best account.**

The sheets with no answers here are not mistakes. Anything asking you to explain, design, predict or justify is judged on whether the reasoning holds together — not on matching a particular sentence.

Grown-ups: in Ontario's four achievement categories the open sheets assess **Thinking** and **Application**, which is where Level 4 lives. Accuracy alone is Level 3. Growing Success, 2010